

For exercises 1-8, verify the identity.

$$1. \csc \theta - \cot \theta = \frac{\sin \theta}{1 + \cos \theta}$$

$$2. 1 - \frac{\cos^2 \theta}{1 + \sin \theta} = \sin \theta$$

$$3. \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta + \sin^2 \theta} = \cot \theta - 1$$

$$4. \frac{1 + \sin \theta}{\sin \theta} + \frac{\cot \theta - \cos \theta}{\cos \theta} = 2 \csc \theta$$

$$5. \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$6. \sin\left(\theta + \frac{3\pi}{2}\right) + \sin\left(\theta - \frac{\pi}{2}\right) = -2 \cos \theta$$

$$7. \frac{\cot \theta - \tan \theta}{\cot \theta + \tan \theta} = \cos 2\theta$$

$$8. \frac{2 \tan \theta}{1 + \tan^2 \theta} = \sin 2\theta$$

Use an addition or subtraction formula to find the exact value of the expression.

$$9. \sin 255^\circ$$

$$10. \tan \frac{11\pi}{12}$$

Use a half-angle identity to find the exact value of the expression.

$$11. \sin 112.5^\circ$$

$$12. \cos \frac{11\pi}{12}$$

$$13. \tan 75^\circ$$

Find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ using the given information.

14. $\cot \theta = \frac{4}{3}$; $\sin \theta > 0$

15. $\csc \theta = \frac{25}{7}$; $\tan \theta < 0$

Find $\sin \frac{\theta}{2}$, $\cos \frac{\theta}{2}$, and $\tan \frac{\theta}{2}$ using the given information.

16. $\cot \theta = \frac{12}{5}$; $180^\circ < \theta < 270^\circ$

17. $\cos \theta = -\frac{8}{17}$; $90^\circ < \theta < 180^\circ$

Find the exact value of the expression. Write your answers in radians when appropriate. Draw a picture for each!

18. $\sin^{-1} \frac{\sqrt{3}}{2}$

19. $\tan^{-1} \frac{\sqrt{3}}{3}$

20. $\cos^{-1} \left(-\frac{\sqrt{2}}{2} \right)$

21. $\cos^{-1} 0$

22. $\sin^{-1}(-1)$

23. $\tan^{-1}(-1)$

24. $\cos \left(\cos^{-1} \frac{2}{3} \right)$

25. $\sin(\sin^{-1} 3)$

26. $\sin^{-1} \left(\sin \frac{2\pi}{3} \right)$

27. $\cos(\tan^{-1} \sqrt{3})$

28. $\cos^{-1} \left(\sqrt{3} \sin \frac{\pi}{6} \right)$

29. $\csc \left(\cos^{-1} \frac{7}{25} \right)$

For exercises 30-33, find all solutions of the equation.

30. $\sin \theta + 1 = 0$

31. $\sqrt{2} \cos \theta + 1 = 0$

For exercises 30-33, find all solutions of the equation [continued].

32. $\csc^2 \theta - 4 = 0$

33. $\tan \theta \sin \theta + \sin \theta = 0$

For exercises 34-41, find all solutions of the equation in the interval $[0, 2\pi)$.

34. $\sec \theta (2 \cos \theta - \sqrt{2}) = 0$

35. $\cos \theta - 1 = 0$

36. $2 \sin^2 \theta - 5 \sin \theta = -2$

37. $\tan^3 \theta + \tan^2 \theta - 3 \tan \theta - 3 = 0$

For exercises 34-41, find all solutions of the equation in the interval $[0, 2\pi)$ [continued]

38. $2 \sin^2 \theta - \cos \theta = 1$

39. $\cos^2 \theta = \cos \theta$

40. $\sin 2\theta = -\sin \theta$

41. $3 \tan^2 \theta - 1 = 0$

Answers:

1. – 8. See online key

9. $\frac{-\sqrt{2}-\sqrt{6}}{4}$

10. $\frac{1-\sqrt{3}}{1+\sqrt{3}}$

11. $\frac{\sqrt{2+\sqrt{2}}}{2}$

12. $-\frac{\sqrt{2+\sqrt{3}}}{2}$

13. $2 + \sqrt{3}$

14. $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$

15. $-\frac{336}{625}, \frac{527}{625}, -\frac{336}{527}$

16. $\frac{5\sqrt{26}}{26}, \frac{-\sqrt{26}}{26}, -5$

17. $\frac{5\sqrt{34}}{34}, \frac{3\sqrt{34}}{34}, \frac{5}{3}$

18. $\frac{\pi}{3}$

19. $\frac{\pi}{6}$

20. $\frac{3\pi}{4}$

21. $\frac{\pi}{2}$

22. $-\frac{\pi}{2}$

23. $-\frac{\pi}{4}$

24. $\frac{2}{3}$

25. undefined

26. $\frac{\pi}{3}$

27. $\frac{1}{2}$

28. $\frac{\pi}{6}$

29. $\frac{25}{24}$

30. $\frac{3\pi}{2} + 2k\pi$

31. $\frac{3\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi$

32. $\frac{\pi}{6} + k\pi, \frac{5\pi}{6} + k\pi$

33. $0\pi + k\pi, \frac{3\pi}{4} + k\pi$

34. $\frac{\pi}{4}, \frac{7\pi}{4}$

35. 0π

36. $\frac{\pi}{6}, \frac{5\pi}{6}$

37. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{3\pi}{4}, \frac{7\pi}{4}$

38. $\frac{\pi}{3}, \frac{5\pi}{3}, \pi$

39. $0\pi, \frac{\pi}{2}, \frac{3\pi}{2}$

40. $0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

41. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$